Title: The polynomial automorphism group

Subtitle: Polynomial automorphisms over finite fields, and locally finite polynomial maps.

Abstract:

A polynomial map in this talk is a map $k^n \longrightarrow k^n$ (where k is a field) given by polynomials. For example, $F = (2X + 1 + Y^2, Y + XY)$ is a polynomial map.

The group of polynomial automorphisms on k^n is still not very well understood. In dimension three and up, we cannot even give a concrete list of automorphisms of which we know that they generate the whole group! In this talk I will discuss two topics related to this problem.

Polynomial maps over finite fields: Polynomial automorphisms are mostly studied over $\mathbb{C}$ or $\mathbb{R}$. Rarely they are studied in characteristic p, and hardly ever over a finite field $\mathbb{F}_q$. One of the first questions that one can ask is one can make any bijection $\mathbb{F}_q^n \longrightarrow \mathbb{F}_q^n$ by a polynomial automorphism in dimension n. The answer is quite surprising.

Locally finite polynomial maps: in my opinion, the best guess for an "understandable" set of generators of the automorphism group over $\mathbb{C}$, is the set of locally finite polynomial maps. These are polynomial maps F that satisfy a sort of Cayley-Hamilton characteristic polynomial: i.e. there exists $c_i \in \mathbb{C}$ such that $F^n + c_{n-1}F^{n-1} + \ldots + c_1 F + c_0 I = 0$. I will elaborate a bit on these maps.

In the end, I will link both subjects by conjectures on generating sets for the automorphism group.

The first part of the talk is quite easy and understandable for students who know what a symmetric group is. The second part may be more complicated, but still very well accessible for advanced students.