Annihilating Polynomials of Quadratic Forms

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By applying the Grothendieck construction to the semi-ring of isometry classes over a field K we obtain the *Witt-Grothendieck ring* $\widehat{W}(K)$. The quotient of $\widehat{W}(K)$ by the ideal generated by the hyperbolic plane is called the *Witt ring* $W(K)$. Both $\widehat{W}(K)$ and $W(K)$ are integral (over $\mathbb{Z}$). This means that for every element x of $\widehat{W}(K)$ or $W(K)$ we can find a polynomial $P \in \mathbb{Z}[X]$ such that $P(x) = 0$. We call P an *annihilating polynomial* of x. The study of those annihilating polynomials is quite young and was initiated by David W. Lewis in 1987. We will see that the *annihilating ideal* $\mathrm{Ann}_x \subset \mathbb{Z}[X]$, consisting of all annihilating polynomials of x, carries information about x.

After a short introduction to the algebraic theory of quadratic forms we will define annihilating polynomials and study their anatomy. It will become apparent that for both $\widehat{W}(K)$ and $W(K)$ the signature homomorphisms (and for $\widehat{W}(K)$ also the dimension) have an important influence on the possible form of an annihilating polynomial P. This will be followed by a survey of the known results about and still open questions concerning the factor of P that is independent of those homomorphisms.