Written examination for the course

Random Polymers

Teacher: F. den Hollander

Thursday 19 January 2012, 10:00–13:00, room 401, Snellius

- Write your name and student identification number on each piece of paper you hand in.
- The use of notes and/or textbooks is not allowed.
- The weight of each question is indicated in boldface. The total number of points is 100. *Final score*: ≥ 58 points is sufficient, ≤ 57 points is insufficient.

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(1) (a) **[5]** Give a qualitative description of what a polymer is. Give three possible classifications of polymers.

(b) **[5]** What are the two key ingredients that are needed to define a polymer chain model and how are these ingredients used to define the path measure?

(2) (a) **[5]** Give the definition of the free energy of a polymer chain and explain why the free energy is an important quantity.

(b) **[5]** Describe a possible method to prove existence of the free energy for a polymer chain without disorder. Illustrate this method with an example.

(3) (a) **[5]** Describe the three versions of the path measure for a polymer chain with disorder ω: quenched, average quenched, annealed.

(b) **[5]** Does the disorder typically affect the free energy? What does it mean when the free energy is self-averaging?

(4) (a) **[5]** Define the model of a polymer chain with self-repulsion strength $\beta \in (0, \infty)$ and self-attraction strength $\gamma \in (0, \infty)$.

(b) **[5]** Give a sketch of the conjectured phase diagram in the (β, γ)-plane. Indicate which parts of this phase diagram have been proved and which have not.

(5) (a) **[5]** Define the model of a directed polymer chain interacting with a homogeneous interface through a binding energy $\zeta \in \mathbb{R}$. What is the difference between pinning and wetting?

(b) **[5]** State the theorem identifying the free energy $\zeta \mapsto f(\zeta)$ for the pinning version of this model when the distribution of the length of the excursions away from the interface is arbitrary but recurrent.

(c) **[10]** Give the proof of this theorem.

(d) **[5]** What changes in the Hamiltonian when a force w is added to the endpoint of the polymer, pointing away from the interface? Give the formula for the free energy with force $f(\zeta, w)$ when the polymer can make increments -1 and 1 with probability $\frac{1}{2}p$ each and increment 0 with probability $1-p$, where $p \in (0,1)$. What is noteworthy about this formula?

(6) (a) **[5]** Describe the model of a directed random copolymer chain near a selective interface. Explain the role of the disorder strength $\beta \in (0, \infty)$ and the disorder bias $h \in (0, \infty)$.

(b) **[5]** State the upper and the lower bound that are known for the quenched critical curve $\beta \mapsto h_c^{\mathrm{que}}(\beta)$ of this model.

(c) **[10]** Give the proof of the upper bound via an annealed estimate.

(7) (a) **[5]** Describe the model of a directed polymer chain in a random potential.

(b) **[5]** Explain why $Y_n^{\beta,\omega} = Z_n^{\beta,\omega}/\mathbb{E}(Z_n^{\beta,\omega})$, the ratio of the quenched and the annealed partition sum of length n with disorder ω and disorder strength $\beta \in (0, \infty)$, is a martingale with respect to the natural filtration generated by ω.

(c) **[5]** Explain what weak disorder and strong disorder are.