

# Re-sit Exam Statistics and Probability

February 1, 2013, 13:00-15:00 h.

Include all the R code that you used and attach any figures in .pdf format. It should be clear which code belongs to which problem.

1. Suppose we have a small population:

```
pop <- c(6,7,8,9)
```

- Compute the population mean  $\mu$  and the population variance  $\sigma^2$ .
- Take all possible samples of size  $n = 2$  from this population and for each sample compute the sample mean  $\bar{x}_i$  and the sample variance  $s^2$ .
- Plot the histogram of the sampling distribution of the sample means.
- What do you notice about the values of the sample means  $\bar{x}_i$  computed in (b)?

2. Suppose that  $X_1, X_2, \dots, X_n$  are i.i.d. from a distribution

$$f(x; \theta) = \left( \frac{\theta}{1 + \theta} \right)^x \frac{1}{1 + \theta}$$

where  $\theta > 0$  and  $x = 0, 1, 2, \dots$

- Demonstrate mathematically (with pen and paper) that the maximum likelihood estimator of  $\theta$  is given by

$$\hat{\theta} = \sum_{i=1}^n x_i$$

The following 5 independent observations were taken from this distribution:

```
x <- c(0,1,0,0,1)
```

- Plot the loglikelihood function.
- Maximize the loglikelihood using R.

3. In a double-blind randomized controlled clinical trial 100 volunteers were enrolled in November 2012 and randomly divided into two groups of 50. One group received a flu vaccine and the other group received placebo. Three months later the volunteers were asked if they'd gotten the flu. The results are given in the following two by two table

	flu	no flu
vaccine	31	19
placebo	22	28

We want to test the null hypothesis that there is no difference between the groups at level  $\alpha = 0.1$ .

- Compute the expected numbers in the two by two table under the null hypothesis.
  - Compute the chi squared test statistic.
  - What is the critical value?
  - Do you reject the null hypothesis?
4. Suppose we have a *single* sample  $X$  from the uniform distribution on the interval  $[0, \theta]$ . Recall that this distribution has density function

$$f(x) = \begin{cases} 1/\theta & \text{if } 0 \leq x \leq \theta \\ 0 & \text{otherwise} \end{cases}$$

We want to test

$$H_0 : \theta = 1 \quad \text{versus} \quad A : \theta < 1$$

at level  $\alpha = 0.05$ .

- We reject  $H_0$  if  $X < c$ . Determine  $c$  so that the test has level  $\alpha = 0.05$
- Suppose that  $\theta$  is in fact equal to 0.3. What is the probability of committing a type II error?
- Plot the power function of the test for  $\theta$  between 0 and 2.