TOPOLOGY II-2. EXAM.

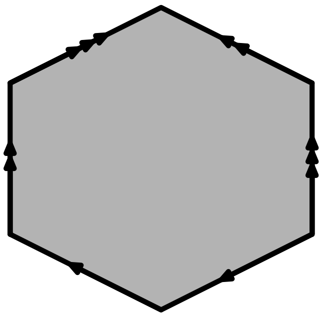
30 MAY, 2007.

Problem 1. Prove that a map $f : S^1 \to S^1$ is homotopic to the constant map if and only if f is a composition of two maps $f = f_1 f_2$, where $f_1 : \mathbb{R} \to S^1$ and $f_2 : S^1 \to \mathbb{R}$.

Problem 2. Consider the quotient space $S^1 \times S^1/(x, y) \sim (y, x)$. Prove that this space is homeomorphic to the Möbius band.

Problem 3. Prove that the Klein bottle from which a point is deleted is homotopy equivalent to the wedge product of k circles and find k.

Problem 4. The sides of the octagon are identified in accordance with the arrows:

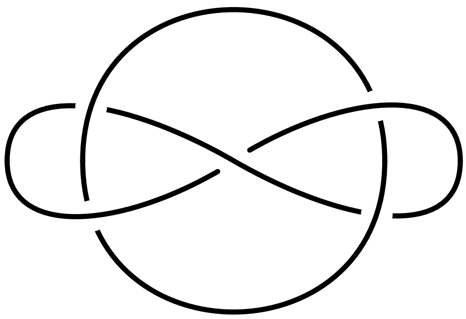


Determine the surface thus obtained.

Problem 5. Calculate $\pi_1(\mathbb{R}P^n)$, for $n = 1, 2, 3, \ldots$

Problem 6. [x2 credit question] Find all Betti numbers of $S^2 \times \mathbb{R}P^2$.

Problem 7. Prove that the Whitehead link



is not trivial.

Problem 8. [Extra credit question] Find a universal covering of the 'dunce hat'.

Date: 30 May, 2007.