The support problem for the integers

Pál Erdős in 1988 asked the following question:

Let x and y be positive integers with the property that for all positive integers n the set of prime numbers dividing $x^n - 1$ is equal to the set of prime numbers dividing $y^n - 1$. Is then $x = y$?

This problem was solved by Corrales-Rodrigáñez and Schoof in 1997: the answer is affermative!

In this talk we will sketch the proof of this result, which is based on Kummer theory. According to time, we will speak about generalizations of Erdős's question.

It is an easy exercise to prove that the set of prime numbers dividing x is equal to the set of prime numbers dividing y (hint: use Fermat's Little Theorem). Also remark that the condition in Erdős's question is equivalent to the following: for every prime number p either p divides x and y or the order of $x \pmod{p}$ in $\mathbb{F}_p^*$ equals the order of $y \pmod{p}$ in $\mathbb{F}_p^*$.

The support of an integer a is the set of prime numbers p such that $a = 0 \pmod{p}$. If $a \neq 0$ then the support of a is simply the set of prime divisors of a. This definition explains why Erdős's question was actually called the 'support' problem.

Remark that if $a > 0$ then the radical of a is the product of the primes lying in the support of a. Then it is not surprising that the abc-conjecture implies also the support problem.

References

[1] C. Corrales-Rodrigáñez and R. Schoof, *The support problem and its elliptic analogous*, Journal of Number Theory, 1997, vol.64 n.2, pp. 276-290