Let $\mathbb{F}$ be a finite field of characteristic $\ell > 0$, F a number field, G_F the absolute Galois group of F and let $\bar{\rho} : G_F \to \mathrm{GL}_N(\mathbb{F})$ be an absolutely irreducible continuous representation. Suppose S is a finite set of places containing all places above ℓ and above ∞ and all those at which $\bar{\rho}$ ramifies. Let $\mathcal{O}$ be a complete discrete valuation ring of characteristic zero with residue field $\mathbb{F}$. In such a situation one may consider all deformations of $\bar{\rho}$ to $\mathcal{O}$-algebras which are unramified outside S and satisfy certain local deformation conditions at the places in S. This was first studied by Mazur, and under rather general hypotheses, the existence of a universal deformation ring was proven.

It turns out to be useful to represent universal deformation rings as quotients of power series ring over $\mathcal{O}$ by suitable ideals I. The talk will present some results on the number of generators needed for an ideal I in such a presentation. These results are among the (many) tools used in the recent attacks on Serre's conjecture by C. Khare and others, which yielded a complete proof of Serre's conjecture in the level one situation. Recently M. Kisin considered presentations of global deformation rings as quotients of power series rings over R_{loc}, where R_{loc} is made up of local (versal) deformation rings. If time permits, I shall try to explain some of this, too.