Exercises for February 20

1. (Graded) Let $Q := V(X_0X_3 - X_1X_2)$ in $\mathbb{P}^3$, and let P be in Q. Show that $T_PQ \cap Q$ consists of 2 distinct lines. Hint: write $P = (a_0 : a_1 : a_2 : a_3)$ and show that by symmetry one can assume that $a_0 = 1$. Then work in the affine part $D(X_0) = \mathbb{P}^3 - V(X_0)$ and do your computations there.

2. Let A be the $\mathbb{R}$-algebra of continuous functions $f\colon \mathbb{R} \to \mathbb{R}$. Let m be the subset of A consisting of f such that $f(0) = 0$. Show that m is a maximal ideal of A, and that $m = m^2$. Hint: first consider f in m such that $f(x) \geq 0$ for all x; for the general case it may be convenient to consider complex numbers.

3. Let n and m be positive integers. Let $f = (f_1, \ldots, f_m)$ be a polynomial map from $\mathbb{A}^n$ to $\mathbb{A}^m$. Show that the tangent maps $T_Pf\colon T_P\mathbb{A}^n \to T_P\mathbb{A}^m$ as defined in the lecture are given by the usual formulas.