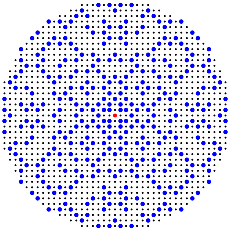
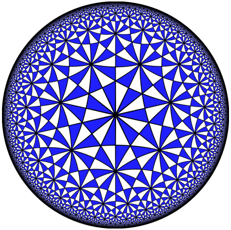
Langlands in the Lowlands

Welke symmetrieën schuilen er onder priemgetallen en hoe kan de wiskundige getaltheorie bijdragen aan natuurkundige theorievorming?

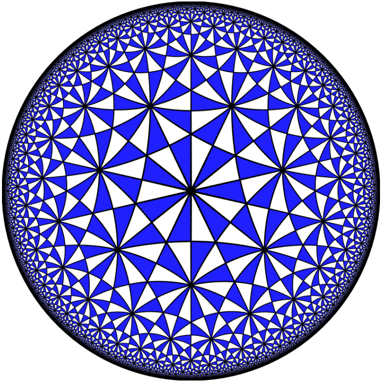


DIAMANT
Number Theory,
Discrete Mathematics, Algebra,
(post-quantum) Cryptology



GQT
Representation Theory,
Geometry, Topology,
Mathematical Physics

Symmetries in geometry and in analysis



Harmonic analysis with boundary conditions of arithmetic origin.

For experts: $\mathrm{SL}_d(\mathbb{R})/\mathrm{SL}_d(\mathbb{Z})$.

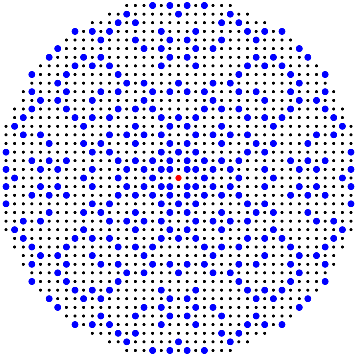
The eigenfunctions form a discrete spectrum.

Infinite dimensional unitary representation theory of $\mathrm{SL}_d(\mathbb{R})$.

Modular forms.

$$q\prod_{n\geq 1}(1-q^n)^2(1-q^{11n})^2=\sum_{n\geq 1}a_nq^n$$

Symmetries in number theory, prime numbers



Galois symmetry:

$$\sigma(a+bi) = a-bi,$$
$$\sigma(a+b\sqrt{2}) = a-b\sqrt{2}.$$

Primes and Galois symmetry give generating functions.

Elliptic curve: $y^2+y-x^3+x^2=0$

$$b_p := p - \#\{(x,y) : 0 \le x, y < p,\ p \text{ divides } y^2+y-x^3+x^2\}$$

$$L(s) = \left(1-11^{-s}\right)^{-1} \cdot \prod_{p \neq 11} \left(1 - b_p p^{-s} + p \cdot p^{-2s}\right)^{-1}$$

Langlands's daring conjecture

Langlands (1967): both sides should produce *the same* generating functions!

Modularity of $y^2 + y - x^3 + x^2 = 0$:

symmetry in number theory	$L(s) = \sum_{n \geq 1} a_n n^{-s}$	symmetry in analysis

Enormous benefit *when* true:

analytic properties of $L(s)$ $+$ *arithmetic information on a_p.*

Wiles (1994, Abel Prize 2016): Fermat's Last Theorem, via Langlands for elliptic curves, "opening a new era in number theory."