2009

Euclides vs. Origami

Trisecting an angle with origami

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16-12-2009

CONTENTS

INTRODUCTION

Today's mathematics contains many insolvable and seemingly impossible problems. A display of what keeps scientists busy are The seven Millennium Prize Problems. Of course, there are still many more problems and developments which are investigated.

In discussion with our mentor we decided to elucidate on of the three classical problems; the trisection of an angle. Our research will cover the possibilities as well as the limitations of this trisection. The focus will not be on the trisection of an angle according to Euclidian mathematic though. We will be analyzing this problem in another way. We have chosen to do this by using origami. Origami makes it very attractive to link algebra with geometry. Plus, it is not a conventional way of analyzing math.

Our goal is to understand why it is not possible to trisect an angle with a compass and straightedge, while it is possible to do so with origami. We explicitly choose to try to understand and not to prove why the trisection is not possible with a compass and straightedge, since knowledge is required we do not have and simply are not able to understand yet.

First we will focus on the principles of origami, for instance the folding axioms. After understanding these principles, we will pass on to mathematical proves which will enable us to understand the trisection of an angle.

The possibilities of a single piece of paper will undoubtedly stagger you.

CHAPTER 1: SIMPLE CONSTRUCTIONS

The Japanese art of paper folding, origami, is known for its multiple possibilities of displaying animals, flowers, fruits and vegetables, even Christmas and Halloween related designs. This is not its only property, it is closely related to mathematics, geometry in particular. It would have been unimaginable to solve Classical Euclidian problems, but origami makes many more possible. An example of one of those problems is the trisection of an angle. Further discussion about the trisection will be found in chapter 4.

First of all, certain mathematical ‘rules’ should be defined in order to make geometric construction with origami. Euclidian constructions consist of points which will be used together with a pair of compasses and a straightedge to draw lines and circles. Points of intersection will arise which will be used for any further Euclidian construction.

However, it works different for origami. In this case, the creases are of great importance since these are actually lines. At the point where two creases intersect, an point of intersection is formed. It sounds very logical, but even the art of paper folding has its own rules when used in geometry. At this moment, there are seven axioms for origami discovered by the Japanese mathematician Humiaki Huzita. The first five axioms are formulated as following (see also figure 1)[1]:

1) *Given two points p_1 and p_2, there is a unique fold that passes through both of them.*
2) *Given two points p_1 and p_2, there is a unique fold that places p_1 onto p_2.*
3) *Given two lines l_1 and l_2, there is a fold that places l_1 onto l_2.*
4) *Given a point p_1 and a line l_1, there is a unique fold perpendicular to l_1 that passes through point p_1.*
5) *Given two points p_1 and p_2 and a line l_1, there is a fold that places p_1 onto l_1 and passes through p_2.*

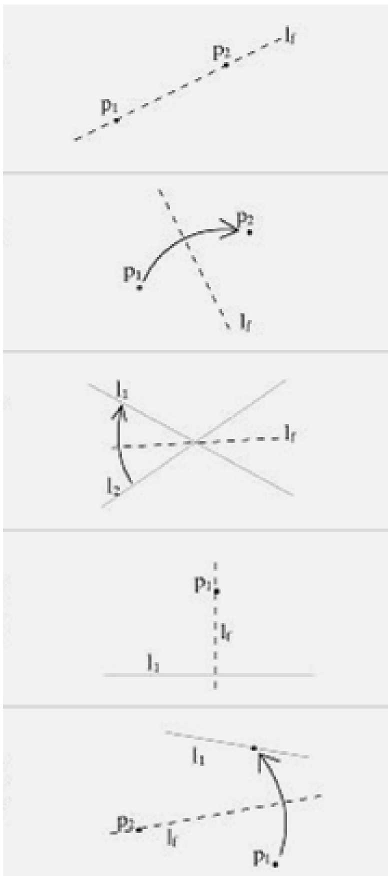

FIGURE 1

[1] "Understanding Geometry through Origami Axioms" in the *Proceedings of the First International Conference on Origami in Education and Therapy (COET91)*, J. Smith ed., British Origami Society, 1992, pp. 37-70

Many constructions can be made with just these five axioms. But it is of great importance to understand the system behind the folding. Therefore one of the most fundamental pieces in mathematics, namely the Cartesian coordinate system, will be first contemplated before the seventh axiom will be considered. The invention of the Cartesian coordinate system by René Descartes in the seventeenth century had a significant influence on mathematics by having linked Euclidian geometry and algebra.

It is known that all real numbers can be found on a line within a plane; the real line. When these numbers are points on the coordinate system, it is possible to calculate with origami. In the following example it is shown how origami is used to add, subtract, multiply and divide. Naturally, this will be done step by step. Each axiom we used will be indicated behind each step.

1) *Draw two points, A and B. Give point A the coordinates (0,0) and point B (1,0).*

2) *Make a crease through points A and B, an x-axis is created.* (A1)

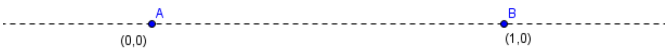

FIGURE 2

3) *Make a crease through point A, such that the x-axis is on top of itself, an y-axis is created.* (A4)

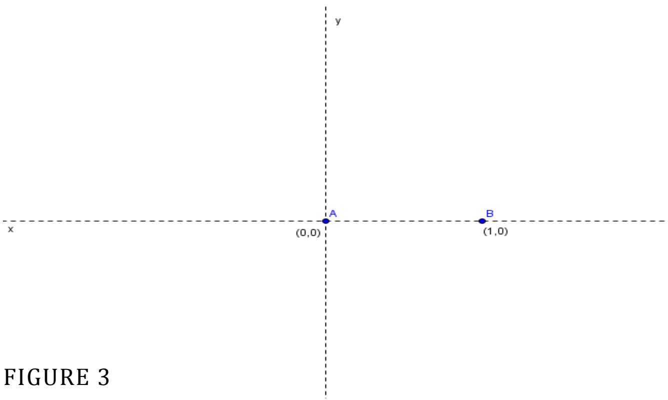

FIGURE 3

4) *With these points and axes, the line y=x can be folded by making a crease through the origin and folding the positive x-axis exactly on the positive y-axis.* (A3)

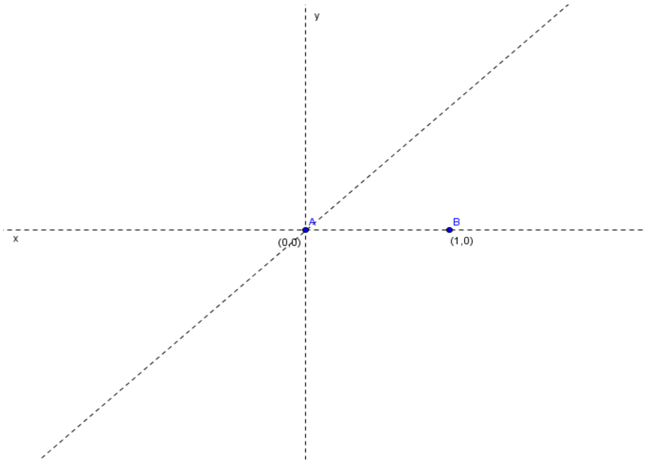

FIGURE 4

5) *Make a crease through point B, such that the x-axis is on top of itself (the same method as in folding the y-axis, see step 3). A point of intersection can be found which is point(1,1).* (A4)

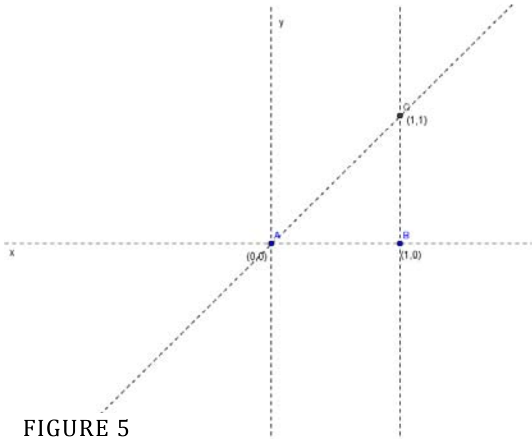

FIGURE 5

6) *By understanding the methods we use for folding these axes and points, more points like (0,1), (-1,1), (-1,0) can be created.* (A1-4)

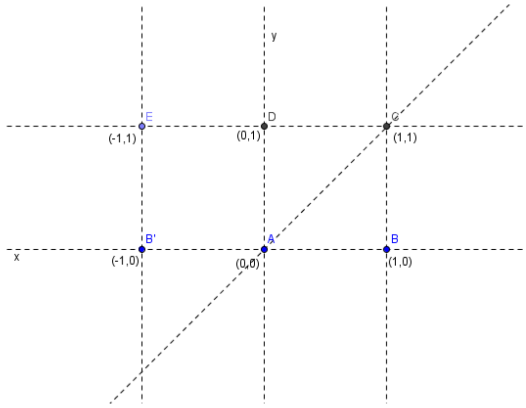

FIGURE 6

7) *The coordinates can also contain fractions like* ($\frac{1}{3}$; 0)*, which is a bit more complicated.*
Point ($\frac{1}{3}$; 0) *can be found by finding an intersection of a crease through point (0,0) and(1,3).* (A1)
Those are not the only points of intersection, another one can be found on the crease y=1, which gives ($\frac{1}{3}$; 1)*.*
A vertical line through this point will intersect the x-axis in point ($\frac{1}{3}$; 0)*.* (A4)

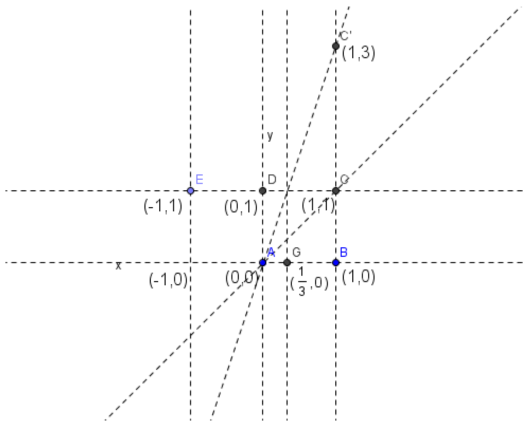

FIGURE 7

Now the basics of geometrical folding are understood, a bit more complex example with the coordinates (a,b) will follow. Point (a,b) is given and will be used to show that additions, subtractions, multiplications and divisions can be folded.

1) *Create a coordinate system according to the previous step 1 to 3.*

2) *(a,0): Make a crease through point (a,b), such that the x-axis is on top of itself. (A4) Point (a,0)is the point of intersection of the new fold and the x-axis.*

3) *(0,b): Make a crease through point (a,b), such that the y-axis is on top of itself.(A4)*

4) *(a+b,0): Make a diagonal crease from the top left to the bottom right through point (a,b). This diagonal can be created when the negative side of the crease: x=a is exactly on the positive side of the crease: y=b. Make sure that the diagonal crease runs through point (a,b). The intersection of the x-axis with the diagonal is the point (a+b,0).*
(A5: l_2 is y=b, p_2 is point (a,b), l_r will be the diagonal and p_1 will be (a+b,0).)

5) *(ab,0): Choose a point (1,b) with the x-coordinate 1. Make a crease: y=bx (the line through (0,0) and (1,b)). (A1)*
The intersection of crease: y=bx with crease: x=a is point (a,ab). Next, make a crease through this points, such that the y-axis is exactly on top of itself(A4).
The intersection with the x-axis gives point (ab,0)

6) *(1⁄b,0): First, create a line through (0,0)and (a,b).* (A1)
This line will also intersect the line y=1, the coordinates of this point are ($\frac{1}{3}$; 1).
A vertical line through point ($\frac{1}{3}$; 1) will intersect the x-axis in point ($\frac{1}{3}$; 0). (A4)

What should be noticed is the system of folding. By using only the first five axioms, many constructions are possible. All rational numbers can actually be found. It is like a simple calculator which has only buttons to add, subtract, multiply and divide.

CHAPTER 2: SECOND DEGREE FOLDING

The comparison with a calculator was already mentioned in the previous chapter. In this chapter two more imaginary buttons will be added; the square root function and the squaring function. To continue with folding while using the extra functions, a new axiom needs to be introduced. This will be the seventh and can be formulated as following (see also figure 8)[2]:

7) Given a point p_1 and two lines l_1 and l_2, we can make a fold perpendicular to l_2 that places p_1 onto line l_1.

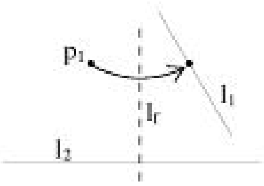

FIGURE 8

Axiom seven requires a folding line with a second degree equation. To demonstrate this, the same axes as in chapter one are necessary. Figure 8 shows that a point p_1 and line l_1, both arbitrarily chosen, are also necessary. Suppose the coordinates of p_1 are (0,1) and the expression for l_1 is y=-1.

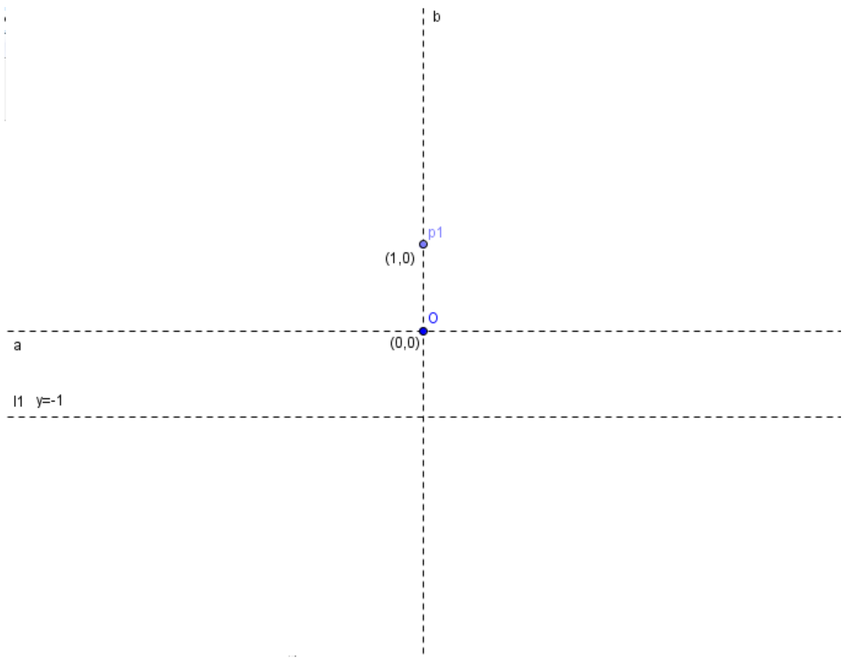

FIGURE 9

[2] "Understanding Geometry through Origami Axioms" in the *Proceedings of the First International Conference on Origami in Education and Therapy (COET91)*, J. Smith ed., British Origami Society, 1992, pp. 37-70

According to axiom seven, p_1 has to be exactly on top of l_1, such that its folding line (l_r) is perpendicular to a yet unknown line l_2. The point where p_1 is on top of crease y=1, is called p_2 and has the coordinates (a,-1). (Note that 'a' stands for any random rational number.)

The line l_r has to be perpendicular to l_2. Therefore, when l_2 runs through p_1 and p_2, the formula of l_2 is $y=-2 \cdot \frac{1}{a}+1$ ('a' shift from p_1 to p_2 is 'a' to the left and 2 down). Hence the formula of any point on the line l_2 is:

(x.y)=0,1+t(a,-2)

i.e.

(x(t),y(t))=0,1+t(a,-2)

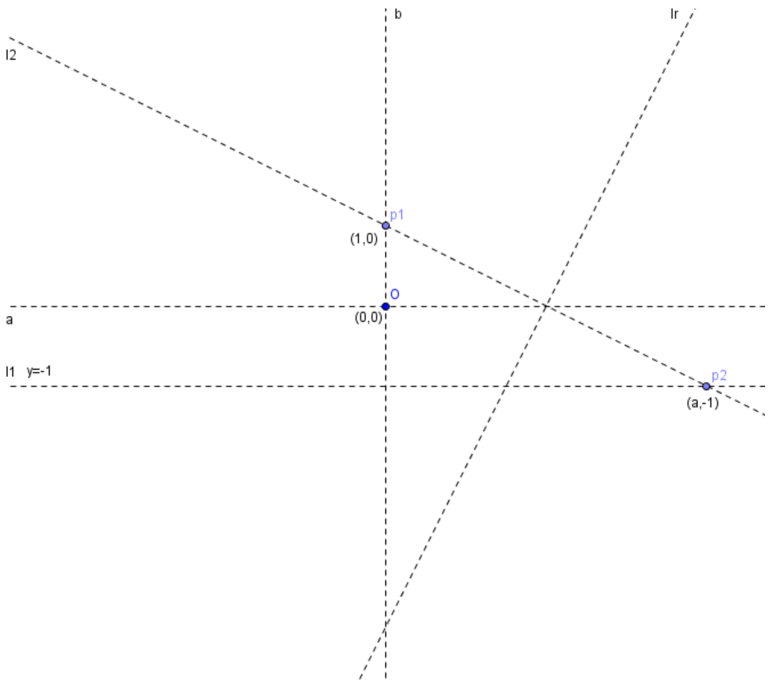

FIGURE 10

The point q_1 divides the line segment p_1 and p_2 exactly in two equal pieces. The coordinates of q_1 will be found when t = $\frac{1}{2}$.

$$(x(\tfrac{1}{2}), y(\tfrac{1}{2}))=(\tfrac{1}{2}a, 0)$$

The point of intersection of l_r and l_2 is at the point q_1 $(\frac{1}{2}a, 0)$. But l_r has a more important point of intersection with the y-axis. Its coordinates contain a squaring, and that is the key of this prove. First, the formula of l_r needs to be contrived get the coordinates of q_2. The formula will be equal to '0' because q_2 is located on the y-axis. The fundamental part of the formula is the slope, which can be facilely found.

$$f'\cdot g'=-1$$

i.e.

$$l_r' \cdot l_2'=-1$$

The slope of l_r can be easily derived with a=dy/dx.
When you fill this in, this gives: a= $\frac{-2}{a}$. It is now possible to find the slope of the crease:

l_2: $y = -2 \cdot \frac{1}{a} + 1$ → $(2a^{-2})$
$l_2' = -1 \cdot a/-2$
$l_2' = a/2$

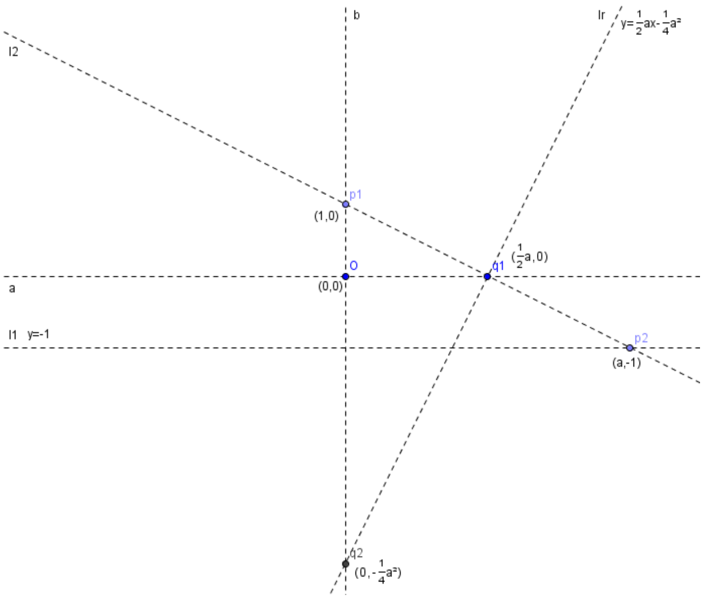

FIGURE 11 (An enlargement is shown on page 11 , figure 12)

It is now possible to formulate the equation of the fold:

Given: point q_1 $(\frac{1}{2}a, 0)$

k: y=ax+b

Calculated :

$$\left.\begin{aligned} 0 &= \frac{a}{2}\left(\frac{1}{2}a\right) + b \\ b &= \frac{1}{4}a^2 \end{aligned}\right\} \quad k\text{: } y = \frac{1}{2}ax - \frac{1}{4}a^2$$

This gives us the coordinates of point q_2, namely $(0,-\frac{1}{4}a^2)$. When giving a closer look to the coordinates of q_1 and q_2 though, something striking will draw your attention. Point $q_2(0,-\frac{1}{4}a^2)$ and point $q_1(\frac{1}{2}a,0)$. The y-coordinate of point q_2 is the square of the x-coordinate of point q_1. Of course you'll have rectify with '-a', since point q_2 intersects the negative y-axes. Contrary one can conclude that the x-coordinate of point q_1 is the root of the y-coordinate of point Q. This means that point Q is the reflection of point P. Therefore it is indeed possible to square and to extract roots with origami.

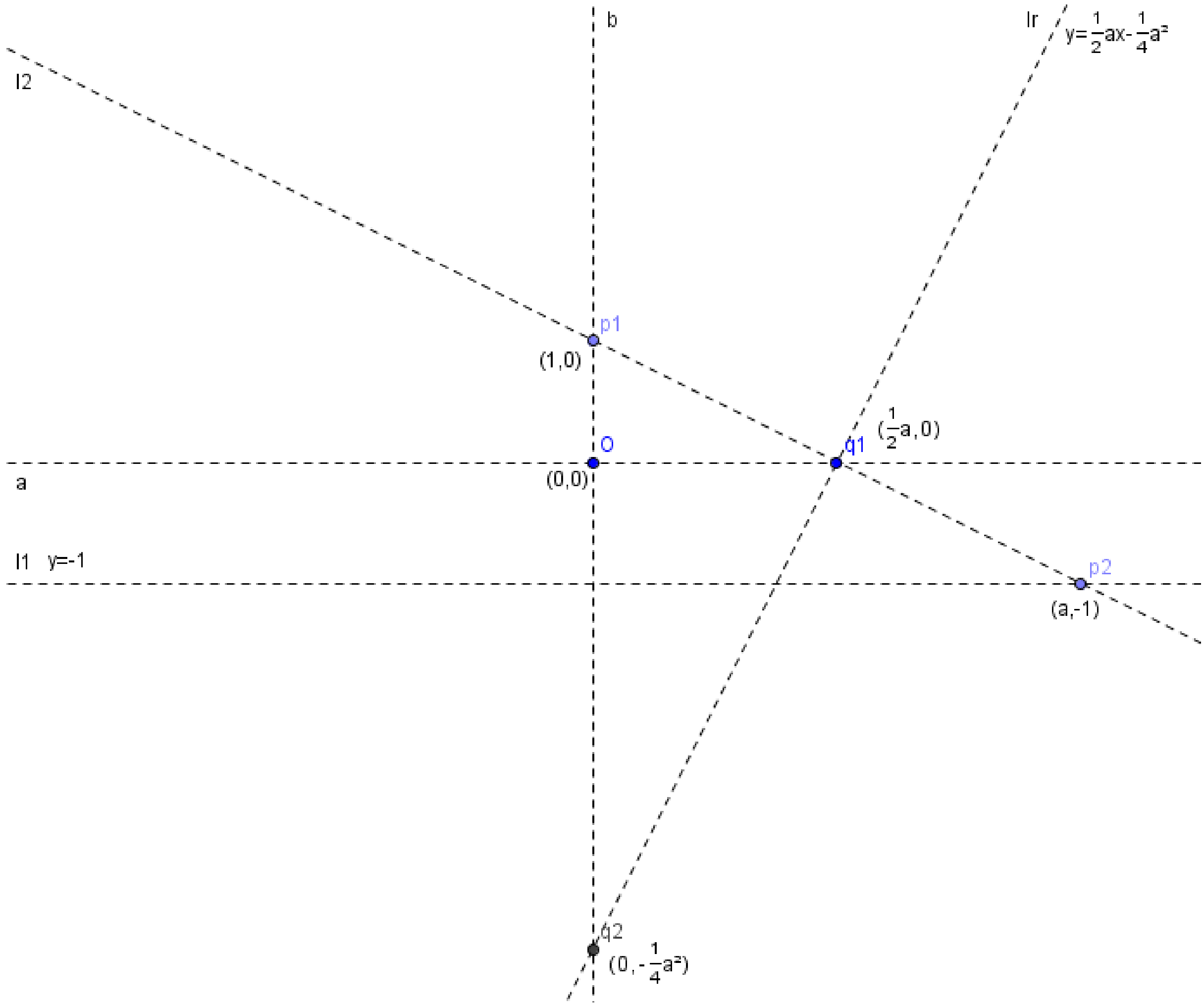

FIGURE 12

CHAPTER 3: THIRD DEGREE EQUATIONS

The following example will be used to prove that it is possible to use origami to solve third degree equations. By giving a proof we will continue to show the difference in third degree equations with origami and with a ruler and a pair of compasses. In this proof axiom number six will be used. This axiom is described below and in figure 13.

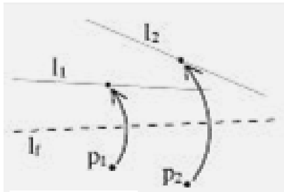

FIGURE 13

6) *Given two points p_1 and p_2 and two lines l_1 and l_2, there is a fold that places p_1 onto l_1 and p_2 onto l_2.*

We will start with the following two things in a coordinate system:

Fold line: $F = \frac{1}{2}ax - \frac{1}{4}a^2$
Point $P_1(0,1)$

The location of point $P_2(s,t)$ (note that 's' and 't' stand for any random rational number) needs to be worked out. P_2 This is accomplished by using a folding line to reflect P_1 into P_2 in the direction of (-a,2). P_1 and P_2 are together on a line which is perpendicular to 'F' and runs through P_1. Its slope will be $-\frac{1}{2}a$.

The coordinate (-a,2) needs to be multiplied by a certain number to find the exact location of P_2, we call this number 'λ'.

The next step is to find λ with (s,t)+λ(-a,2). Written as one, gives (s-λa,t+sλ) which stands for the x- and y-coordinates.

Point P_2 is exactly on 'F' when F = y_{P2}. In order to solve this, we need to fill the coordinates (s-λa,t+sλ), which gives the following answer:

$t+2\lambda=\frac{1}{2}a\,(s-\lambda a)-\frac{1}{4}a^2$

When solved, it will give us the answer of λ:

$\lambda(2+\frac{1}{2}a^2)= -t-\frac{1}{4}a^2+\frac{1}{2}a\,s$

So $\lambda=(\frac{1}{2}a\,s-t-\frac{1}{4}a^2)/(2+\frac{1}{2}a^2)=(2as-4t-a^2)/(8+2a^2)$

The location of P_2 is: $(s,t)+(2\lambda(-a,2)) = (s-2\lambda a, t+4\lambda)$

When we introduce 'w' with $w=s-2\lambda a$, we can replace 2λ in this equation for:

$$2\lambda=2(2as-4t-a^2)/(8+2a^2)=(2as-4t-a^2)/(4+a^2)$$

Therefore 'w' will be:

$$w=s-a((2as-4t-a^2)/(4+a^2))$$

$$w(4+a^2)=s(4+a^2)-2a^2s+4at+a^3$$

When we rewrite this equation as an equation of a, it will give us a third degree equation:

$$a^3+a^2(-2s+s-w)+a(4t)+4s-4w=0$$

or even more simple

$$a^3+a^2(-s-w)+a(4t)+4(s-w)=0$$

By this evidence we can therefore say that it is likely to trisect an angle with origami because it is possible to solve third degree equations by folding a paper. This can be illustrated by a simple example. We'll start with the equation we found:

$$a^3+a^2(-s-w)+a(4t)+4(s-w)=0$$

What one has to conclude in order to be able to solve third degree equations, is that if c_2, c_1 and c_0 are given figures, you'll be able to choose s, t and w so that:

$$\left.\begin{array}{l} -s-w=c_2 \\ 4t=c_1 \\ 4(s-w)=c_0 \end{array}\right\} \quad \begin{array}{l} s=\frac{1}{8}c_0-\frac{1}{2}c_2 \\ t=\frac{1}{4}c_1 \\ w=-\frac{1}{8}c_0-\frac{1}{2}c_2 \end{array}$$

If we fill this in the equation we found, we'll have a more simple equation:

$$a^3+c_2a^2+c_1a+c_0=0 \rightarrow x^3+c_2x^2+c_1x+c_0=0$$

Let's say that we have the equation *x^3-2=0.* This means that c_2=0, c_1=0 and c_0=2. That gives s=-$\frac{1}{4}$, t= 0 and w=$\frac{1}{4}$. We've found the coordinates of 'F', namely x=$\frac{1}{4}$, the coordinates of P_2 are (-$\frac{1}{4}$,0). Now we have to fold the point P_2 onto 'F' in a way that P_1 will be fold simultaneously onto F. If this is accomplished, you will find the point where P1 overlaps F. In our case you will find ≈ (1,26;-1). This is not a coincidence. We started out with our simple equation: x^3-2=0. It's obvious that the solution to this equation is x=$\sqrt[3]{2}$. This is approximately 1,26. The x-coordinate we found is also 1,26.

As you can see it is not that hard to solve third degree equations with origami. With this method of working it is possible to even more difficult equations in a relatively easy way, especially when you forget to bring your graphic calculator. A piece of paper will do!

CHAPTER 4: TRISECTING AN ANGLE WITH ORIGAMI

Now we know how third degree equations are to be solved with origami, we are able to embark on the trisection of an angle with origami. Again this will be done by giving points coordinates, instead of using a method purely based on geometry.

We'll start out with a unit circle with radius 1. Since we want to trisect an angle, we'll arbitrarily choose a point B on this circle with coordinates (cos(3α),sin(3α)). The size of the corresponding angle will be of course '3α'. The doubling formulas that correspond with angles of '2α' are already known, namely:

$Sin(2\alpha)= 2sin(\alpha)cos(\alpha)$
$Cos(2\alpha)=cos^2(\alpha)-sin^2(\alpha)$
$Cos(2\alpha)=1-2sin^2(\alpha)$
$Cos(2\alpha)=2cos^2(\alpha)-1$

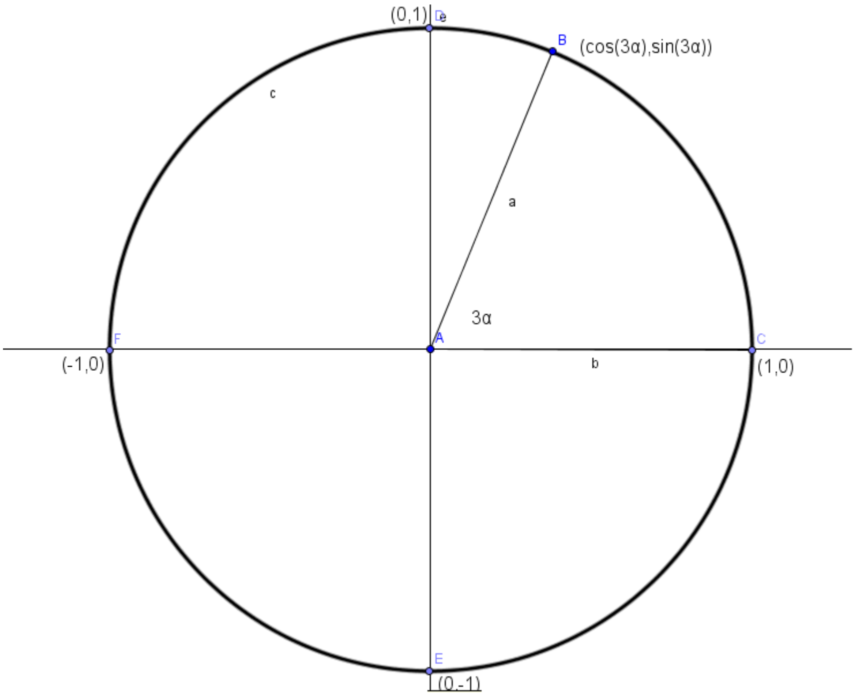

FIGURE 14

Doubling formulas corresponding with 3α angles also exist though, namely:

$Cos(3\alpha)=cos(\alpha+2\alpha)=4cos^3(\alpha)-3cos(\alpha)$
$Sin(3\alpha)=sin(\alpha+2\alpha)=sin(\alpha)(4cos^4(2\alpha-1)$

The first formula is the most interesting, namely $cos(3\alpha)=cos(\alpha+2\alpha)=4cos^3(\alpha)-3cos(\alpha)$. As you can see this is a third degree equation in cos(α). In the previous chapter we proved that it is possible to solve these equations with origami.

Let's say that our angle has a size of $\frac{1}{3}\pi$. De coordinates of point B will be $(\cos(\frac{1}{3}\pi),\sin(\frac{1}{3}\pi))$, i.e. $(\frac{1}{2},\frac{1}{2}\sqrt{3})$. In order to calculate the size of angle α, we review the equation $a^3+a^2(-s-w)+a(4t)+4(s-w)=0$. This means we will have to calculate the unknown a in order to find α. We therefore have the next equation:

$4\cos^3(\alpha)-3\cos(\alpha)-\frac{1}{2}=0.$

Divide in 4 gives

$\text{Cos}^3(\alpha)-\frac{3}{4}\cos(\alpha)-\frac{1}{8}=0.$

It is easy now to find c_2, c_1 and c_0, namely $c_2=0$, $c_1=-\frac{3}{4}$ en $c_0=--\frac{1}{8}$. Since we've already given proof how to solve a third degree equation with origami in the previous chapter, we'll omit the proof in this chapter.

CHAPTER 5: TWO REMAINING UNSOLVED CLASSICAL PROBLEMS

5.1 THE DELIAN PROBLEM – THE DUPLICATION OF A CUBE

Another one of the three classic unsolved problems of antiquity is the duplication of a cube. It is also known as the Delian problem due to a legend concerning the Delians, the citizens of Delos (Greece). They sought for advice from the oracle of Delphi during the time of an assailing pest. They were advised to double the volume of the current altar, which already had the shape of a cube, into a new cube. They thereupon started on doubling their altar and hoped the plague would cease. But no solutions were found and they asked Plato for advice. Plato's interpretation of the oracle's advice was 'that the god was punishing the Greeks for neglecting the study of geometry' . This story is stated in the introduction of Eratoshenes' handbook of mathematics.

By duplicating a cube, of for example volume one, the new cube needs to contain a volume of two. It is necessary that each edge of the new cube has a length of $\sqrt[3]{2}$. It is impossible to solve third degree equations using only a pair of compasses and a straightedge. According to our prove in chapter XX, the Delian problem can be solved by using origami. There are more existing theories which provide the solution of the doubling of a cube, like the cissoid of Diocles, the conchoid of Nicomedes, the Philo line, or the three dimensional construction of Archytas. But non of them is in agreement with the Eucledic axioms.

FIGURE 15

5.2 SQUARING A CIRCLE

Squaring a circle means that the area of a circle is exactly the area of a square. The area of a circle can be calculated with πr^2 . When a circle has radius 1, the area of the circle will be π. Therefore the edges of a square with the same area have to be of a length of $\sqrt{\pi}$. According to the prove of Lindemann in 1882, pi is transcendental. Transcendental means that a number cannot be the root of any non-zero polynomial with rational coefficients. This effectively proved that the construction was impossible with only compass and straightedge.

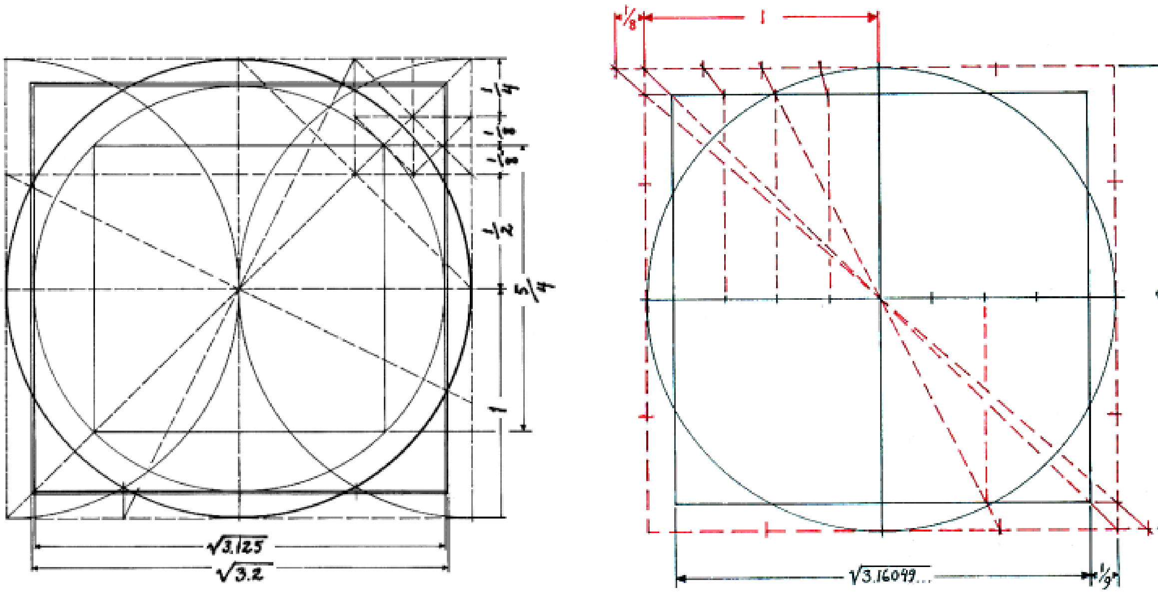

FIGURE 16 *On the left the Babylonian version, on the right the Egyptian Rhind papyrus version, both an attempt to solve the ancient problem.*

CHAPTER 6: EXTRACTION OF ROOTS WITH A COMPASS AND STRAIGHTEDGE

In chapter 2 of part I we proved that it is possible to square with origami. It is interesting to see though how this can be performed by using a compass and a straightedge.

First we'll start by drawing a circle. As you know the equation of a circle is $x^2+y^2=r^2$. The radius of the circle can be chosen arbitrarily, therefore we can say that our radius is r_1. De coordinates of a point B on the circle is (x,y). This situation can be clarified by a simple illustration (figure 17).

In order to find the coordinates of point B, you'll have to make use of the equation of a circle. The x-coordinate of point B can simply be found by making a lead line through point B, intersecting r_1 (figure18).

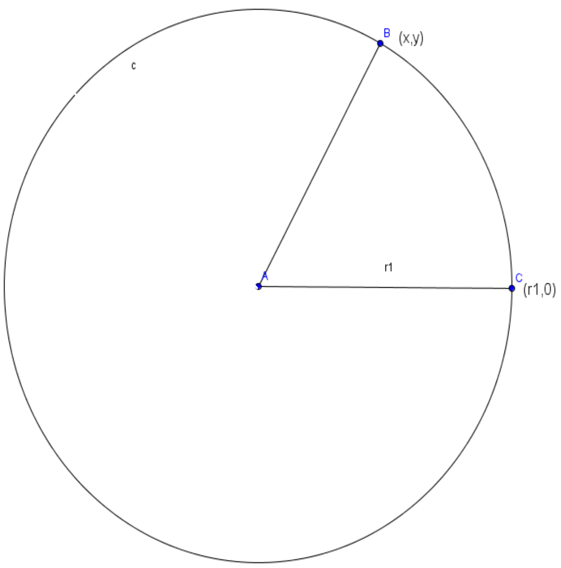

FIGURE 18 $c: x^2+y^2=r^2$

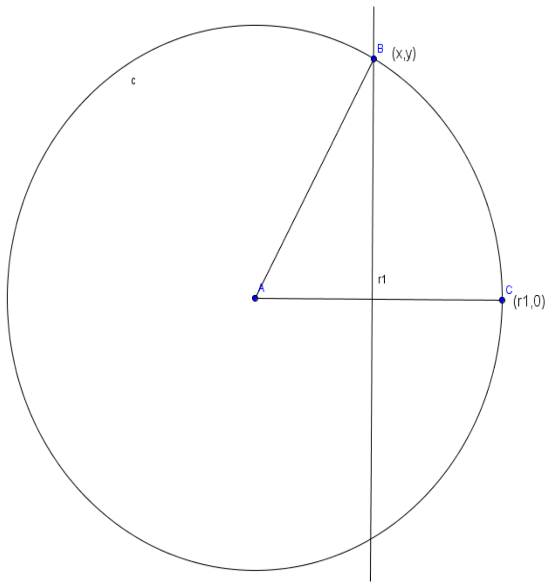

FIGURE 17

One can construct a lead line by starting out with drawing a line and a circle. Subsequently you'll have to construct two other circles with centres on that same line. These circles will also have to enfold the very first circle. One will find two intersections. By drawing another vertical line through these intersections, you'll automatically construct the perpendicular bisector. You can use the same method you used here, to construct a lead line. The only difference is that the circles will have to intersect the point, of which you want to construct the lead line (figure 19 and 20).

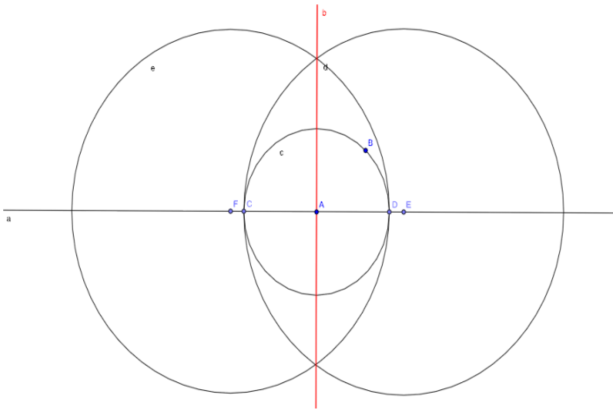

FIGURE 19 *The construction of a perpendicular bisector*

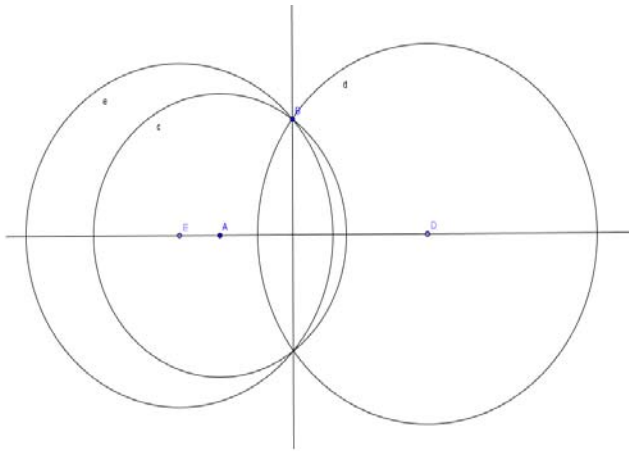

FIGURE 20 *The construction of a lead line*

We are now able to give coordinates to point B an point C, namely C (a+1,0) and B (a-1,y) (figure 21). We are interested in finding out our y-coordinate. We can define our radius by a+1, since it equals the x-coordinate of point C. This information enables us to derive the y-coordinate of point B. If done correctly, one can conclude that squaring and extractions of roots are possible by using a compass and straightedge.

$x^2+y^2=r$

Filling in the x-coordinate of point B and the x-coordinate of point C as the radius gives

$(a-1)^2+y^2=(a+1)^2 \quad \rightarrow \quad a^2-2a+1+y^2=a^2+2a+1$

$y^2=4a \quad \rightarrow \quad y=2\sqrt{a}$

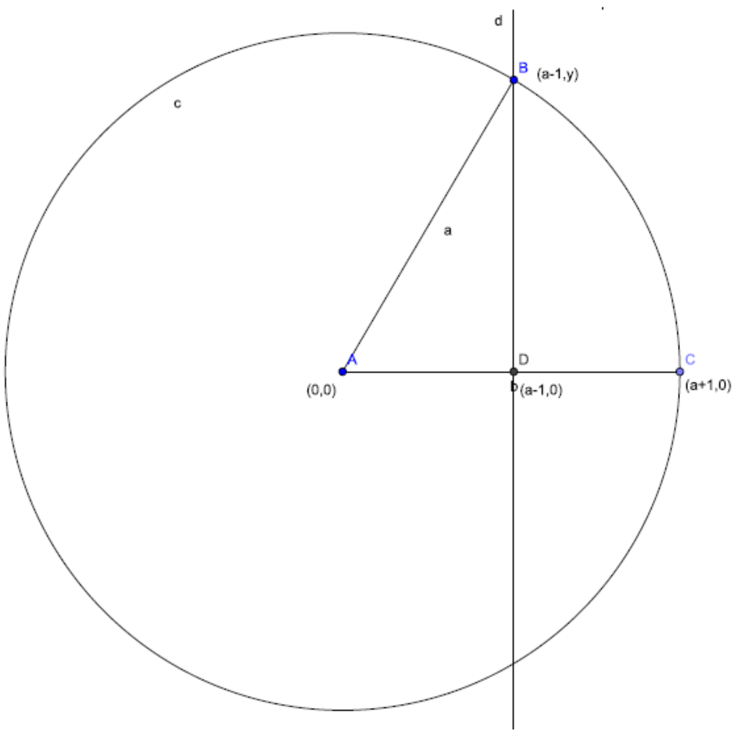

FIGURE 21

How and why did you make the choice to study mathematics after which you decided to specialize in geometry?

Bas Edixhoven

"It all started when I was a sixteen-year old student. My brother went to study electrical engineering at the University of Technology in Delft. Immediately his books on mathematics and physics caught my attention. I came across int. al. very fascinating proves I had not seen before. I came to the conclusion that I had not really been using my brains until that moment. This is what caused me to go study Mathematics and Physics in Leiden."

"I remember reading a article on Falting's work in algebraic geometry in the summer holidays after my third bachelor year. Instantly, I knew that this was in which I wanted to specialize. Two years later I graduated with a doctoral thesis on so called modular curves."

Which position are you currently holding at the Mathematical Institute of Leiden?

"At the moment I am appointed to being the educational director. That means that I'm responsible for the allocation of the classes to the professors. The focus is less on my actual specialism, but this has to be done either. Adjacent to that I guide doctoral students and I also still give classes."

Which project or research are you currently working on?

"As you already know my specialism is the algebraic geometry. I have been working on the complexity of certain problems in computational number theory. What we're trying to do here, is to analyse the number of solutions for a number 'x' in high dimensional spaces."

With the exception of your work at the MI, are you engaged with other activities concerning math?

Bas Edixhoven points to four huge accumulations of A4-sheets, all filled with researches and says: "Being one of the general editors of 'Compositio Mathematica', you are responsible for one pile being published. For the young math-lovers, we have a foundation called 'Vierkant voor wiskunde', of which I am part of the board. I also guide students of the Pre-University."

For which mathematical problem can we wake you up in the middle of the night?

“I will get out of my bed for the solutions to one of the seven Millennium Prize Problems of the Clay Mathematics Institute of Cambridge. These problems came into existence after David Hilbert gave a lecture on unsolved mathematical problems in 1900. So far, only one problem has been solved, namely the Poincare Conjecture.”

What are you the most proud of in your career?

“At present Jean-Marc Couveignes and I are working on a book on the computation of coefficients of a modular form. It will be published as ‘Annals of Mathematics Studies’ by Princeton University Press. I have to say, being the first Dutchman to accomplish such thing at such a prestigious publisher, it has a very special feeling attached to it.”

The ‘Annals of Mathematics Studies’ by Princeton University Press is one of the oldest and most respected series in science publishing, and it has included many of the most important and influential mathematical works of the 20th century. The series continues this tradition into the 21st century as Princeton looks forward to publishing the major works of the new millennium.

CONCLUSION

Our main objective was to understand why it was not possible to trisect an angle. We achieved our goal by starting with analyzing the basics of origami and Euclidian mathematics. These two main subjects were expanded and we proved that both methods can solve second degree equations. When we got to third degree equations though, we found out that it is not possible to solve third degree equations according to the Euclidian axioms for constructions with a pair of compasses and a straightedge.

During our research we developed a different way of thinking. We were forced to be creative and to think outside of the box. Although this does not mean that we did not bump into problems though. Little information on this subject could be found on the internet, which meant that we had to extend a research without really knowing what to look for or what to expect. We also had to deal with an entirely new way of processing information.

It is satisfying to see the end result though. It still dazzles us to see how much is possible with a single piece of paper. We did things we did not think of as possible. We were introduced in the world of math and it fascinated us. By doing this research we were given a little insight in a world we know so little about. We came to the conclusion that mathematics is so much more than we thought it was. That's why it is fair enough to say that we achieved our goal.

ACKNOWLEDGEMENTS

This piece of writing and our changed way of thinking would not exist if we had not the support of certain people. After pages full of math, we found it appropriate to thank those people.

First of all, we would like to thank our mentor, Bas Edixhoven. He accepted two students assigned to him by Pre University College without knowing which pig in a poke he was buying and devoted his precious time to explain the simplest questions. He got assistance from Jinbi Jin, a fourth grade student at the Mathematical Institute of Leiden. Jinbi reviewed our report and gave useful comments.

Although most contact about our research was with these two people, our math teachers at our high schools, Wolfert van Borselen and Dalton The Hague, were always willing to help out if necessary.

Nor should our family be forgotten, they made it possible to go to Leiden at least once a week during a period of thirteen weeks.

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Because of the fact that this piece of work considers a mathematical subject, it contains primarily equations and images created by ourselves with help of mathematicians, who were our main source of wisdom. Our list of sources is therefore limited.

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