Chabauty's method and beyond, by Bas Edixhoven, universiteit Leiden, NL.

Finding <u>all</u> rational solutions of a polynomial equation $f(x,y)=0$ with rational coefficients can be a difficult problem.

An example of this is the case of the so-called "cursed curve", given by the equation:

$$-(y+1)\cdot x^3 + (2y^2+y)\cdot x^2 + (-y^3+y^2-2y+1)\cdot x + (2y^2-3y)\cdot 1 = 0.$$

On this curve, number 1 on the "most wanted list" for a reason that one could also talk about, there were 7 known rational points: $(0,0)$, $(1,0)$, $(-1,0)$, $(0,3/2)$ and the points "at infinity" in the projective plane $(1:0:0)$, $(1:1:0)$ and $(0:1:0)$. For a long time the curve resisted all attackers who wanted to prove that there are no other rational points, whatever tools they brought. The solution came in 2017, published in the Annals of Mathematics in 2019. A team of 5 mathematicians, Balakrishnan, Dogra, Müller, Tuitman and Vonk, had built a new weapon, called "quadratic Chabauty", by implementing the simplest non-linear case of Minhyong Kim's "nonabelian Chabauty" method. In the last 2 years, with Guido Lido, we have made this method much much simpler, the result is now in our preprint on arxiv. In the talk I will address the history of this ancient subject, and try to explain the special role of p-adic numbers (that I will also explain) in Chabauty's method, and what happens in the quadratic case.

<u>Colloquium Mainz.</u>

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