

What is geometry??

Lecture 1, 2011/09/08.

12.

\exists many kinds: kind topology
 examples: differential top./geometry
 analytic geometry \mathbb{C}, \mathbb{R}
 algebraic geometry

functions/equations
 continuous
 differentiable
 analytic
 rational/polynomial

\rightsquigarrow geometric objects, as manifolds, varieties and their morphisms

Common framework: rings spaces, locally rings spaces.

Example: Let X be a C^∞ -manifold, defined ^{in a way} you like.

Then $\forall U \subset X$ open we have $C_X^\infty(U) = \{f: U \rightarrow \mathbb{R} : f \text{ is } C^\infty\}$,
 an \mathbb{R} -sub-als. of $\{f: U \rightarrow \mathbb{R}\}$. And $\forall V \subset U$ incl. of opens we
 have $C_X^\infty(U) \rightarrow C_X^\infty(V), f \mapsto f|_V$.

Finished with: $RSp, LRSp$ were already defined. (Sik too, given local model.)

A a ring. $U \subset \text{Spec } A$ open.

$$\mathcal{O}(U) = \left\{ f: U \rightarrow \coprod_{p \in A} A_p : \right.$$

1. $\forall p \in U, f(p) \in A_p$

2. $\forall p \in U, \exists V \subset U, g, h \in A$ s.t.

(a) $\forall q \in V, h(q) \neq 0$ in $\kappa(q)$

(b) $\forall q \in V, f(q) = \psi_q(h)^{-1} \cdot \psi_q(g)$ in A_q

End of week 1

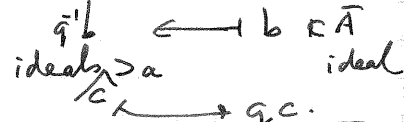
Promised to do: $\mathcal{O}_p = A_p, \mathcal{O}(\text{Spec } A) = A$

Nullstellensatz: ~~...~~ $\forall a \subset A$ ideal, $I(V(a)) = \sqrt{a}$.

Proof of \uparrow . $I(V(a)) = \{f \in A : \forall p \in \text{Spec } A \text{ s.t. } p \supset a, f \in p\}$

$$= \bigcap_{p \supset a} p = q^{-1} \bigcap_{p \in \text{Spec}(A/a)} p \quad (q: A \twoheadrightarrow A/a)$$

Let A be a ring. Then $\bigcap_{\substack{p \subset A \\ \text{prime}}} p = \sqrt{A}$.



Proof \supset clear. Now \supseteq . Let $f \in \bigcap_{p \text{ prime}} p$.
Claim: $A_f = 0$. Proof. Assume not, let $m \subset A_f$ maximal.
 Gives contradiction. \circledast $A_f = 0 \Rightarrow f$ nilpotent. \circledast

$A \xrightarrow{\psi} A_f$

Study $\text{Spec}(\psi)$
 first.