Uitwerkingen werkcollege 6.

4.4.1

4.1.4.(3) $n = 3$, $W = \mathbb{C}^4$, $C = \big((1,1,0,1),(2,0,1,2),(0,-3,-1,1)\big)$.

4.1.4.(4) $n =$, $W = V$, $C = \big(v_1, v_2, v_3\big)$.

4.1.7 $n = 2$, $W = \mathbb{R}^2$, $C = \big((\cos\theta, \sin\theta), (-\sin\theta, \cos\theta)\big)$.

4.1.8 $n = 2$, $W = \mathbb{R}^2$, $C = \big((0,-1),(-1,0)\big)$.

4.1.10 $n = 2$, $W = \mathbb{R}^2$, $C = \big((\frac{3}{5}, \frac{4}{5}), (\frac{4}{5}, -\frac{3}{5})\big)$.

4.1.11 $n = 2$, $W = \mathbb{R}^2$, $C = \big((-\frac{4}{5}, \frac{3}{5}), (\frac{3}{5}, \frac{4}{5})\big)$.

4.4.2 (1) We take $m = n$, $W = F$, and $C = (0, 0, \ldots, 0, 1, 0, \ldots, 0) \in F^n$, with the 1 on the j-th position.

(2) The same: $a = (0, 0, \ldots, 0, 1, 0, \ldots, 0) \in F^n$, with the 1 on the j-th position!

5.3.1

$$(1)\ \begin{pmatrix} 20 \\ -11 \\ -2 \end{pmatrix} \qquad (2)\ \begin{pmatrix} -7 \\ -12 \end{pmatrix} \qquad (3)\ \begin{pmatrix} 1 \\ -12 \\ 3 \\ 5 \end{pmatrix}$$

5.4.2 $m = 3$ and $n = 4$ and

$$w_1 = \begin{pmatrix} -4 \\ 2 \\ 0 \end{pmatrix}, \qquad w_2 = \begin{pmatrix} -3 \\ 2 \\ -3 \end{pmatrix}, \qquad w_3 = \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix}, \qquad w_4 = \begin{pmatrix} -3 \\ -1 \\ -1 \end{pmatrix}.$$

5.4.3 $\begin{pmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{pmatrix}$

5.4.4 (1) $\begin{pmatrix} 3 & 2 & -1 \\ -1 & -1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix}$

(2) $\begin{pmatrix} 1 & 2 & -3 \\ 2 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

(3) $\begin{pmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \end{pmatrix}$

5.5.4 For the first part see Exercise 5.4.1 (the case of Exercise 4.1.7). If $\sigma \colon \mathbb{R}^2 \to \mathbb{R}^2$ is rotation around 0 over an angle β, then for all $v \in \mathbb{R}^2$ we have $\sigma(v) = Bv$ for

$$B = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix}.$$

The composition $\rho \circ \sigma$ is rotation around 0 over $\alpha + \beta$, so on one hand this is given by multiplication with

$$C = \begin{pmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) \end{pmatrix}.$$

On the other hand it is given by multiplication with

$$\begin{aligned} AB &= \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \cdot \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \\ &= \begin{pmatrix} \cos\alpha\cos\beta - \sin\alpha\sin\beta & -\cos\alpha\sin\beta - \sin\alpha\cos\beta \\ \cos\alpha\sin\beta + \sin\alpha\cos\beta & \cos\alpha\cos\beta - \sin\alpha\sin\beta \end{pmatrix} \end{aligned}$$

Comparing the entries of the two equal matrices C and AB, we find the identities given in the exercise.

5.5.10 Je kunt bijvoorbeeld nemen $A = \begin{pmatrix} 1 & 0 \end{pmatrix}$ en $B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. Dan is AB de 1 bij 1 identiteitsmatrix. Dat A niet inverteerbaar is is omdat $f_A \colon F^2 \to F$ niet injectief is ($f_A(e_2) = 0$), en B is niet inverteerbaar omdat $f_B \colon F \to F^2$ niet surjectief is (e_2 zit niet in het beeld, want het beeld is $L(e_1)$).